Anomalous features of the proximity effect in unconventional superconductors

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The transport properties of contacts between unconventional superconductors and normal diffusive metals are analyzed in the framework of the quasiclassical kinetic theory. Using a general boundary condition for the Keldysh-Nambu Green's functions at the S/N interface we calculate the local density of states (DoS) and the voltage-dependent conductance of the contacts. Two cases are considered when S is a spin-singlet d-wave superconductor or a triplet superconductor with $p_x + ip_y$-wave symmetry. We discuss an interplay between the standard proximity effect in a diffusive normal metal and midgap Andreev bound states arising due to internal phase shift at interfaces in unconventional junctions. In the spin-triplet case the pairing amplitude is odd in frequency which is the source of the zero-energy singularity of the local DoS in N and of an anomalous screening of an external magnetic field. The model is further applied to the study of the Josephson effect in SNS junctions based on unconventional superconductors with diffusive N interlayer. The relevance of the results to recent experimental data for unconventional junctions is discussed.

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[1] Y. Tanaka, Y. V. Nazarov, and S. Kashiwaya, Phys. Rev. Lett. **90**, 167003 (2003); Y. Tanaka, Y. V. Nazarov, A. A. Golubov and S. Kashiwaya, Phys. Rev. B **69**, 144519 (2004).
[2] Y. Tanaka, S. Kashiwaya, and T. Yokoyama, Phys. Rev. B **71**, 094513 (2005).
[3] Y. Tanaka, A. A. Golubov, Y. Asano, and S. Kashiwaya, to be published.

Proximity effects in unconventional superconductors

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Collaborators

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- Y. V. Nazarov, Delft ,The Netherlands
- Y. Asano, Hokkaido, Japan

Content of the talk

1. Phase sensitivity of tunneling in unconventional superconductors

2. Ballistic junctions: conductance, Josephson effect, 0- π transitions

3. Anomalies of proximity effect in d- and p-wave diffusive junctions

Symmetry of Cooper Pairs

Pair wavefunction: $$F_{ss'}(\vec{k}) = \langle \hat{c}_{\vec{k}s} \hat{c}_{-\vec{k}s'} \rangle = \underbrace{\Phi(\vec{k})}_{\text{orbital}} \underbrace{\chi(s,s')}_{\text{spin}}$$

totally antisymmetric under electron exchange

$$\vec{k} \rightarrow -\vec{k} \qquad s \leftrightarrow s'$$

even parity $$\Phi(-\vec{k}) = \Phi(\vec{k}) \longrightarrow$$ S=0 singlet

L = 0, 2, 4, 6. . .

s-wave, d-wave, g-wave $\longrightarrow$ **singlet unconventional superconductor**

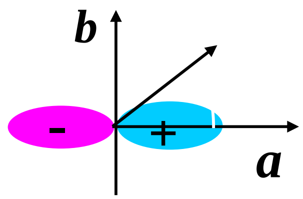
odd parity $$\Phi(-\vec{k}) = -\Phi(\vec{k}) \longrightarrow$$ S=1 triplet

L = 1 ,3 ,5, ...

p-wave, f-wave, h-wave $\longrightarrow$ **triplet superconductor**

Tunneling spectroscopy of Unconventional superconductors

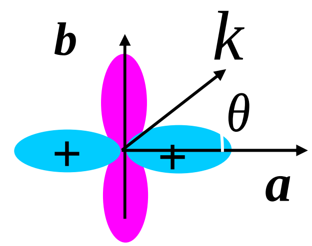
p_x- wave



$\Delta(\theta) = \Delta_0 \cos\theta$

Triplet superconductor

$d_{x^2-y^2}$-wave

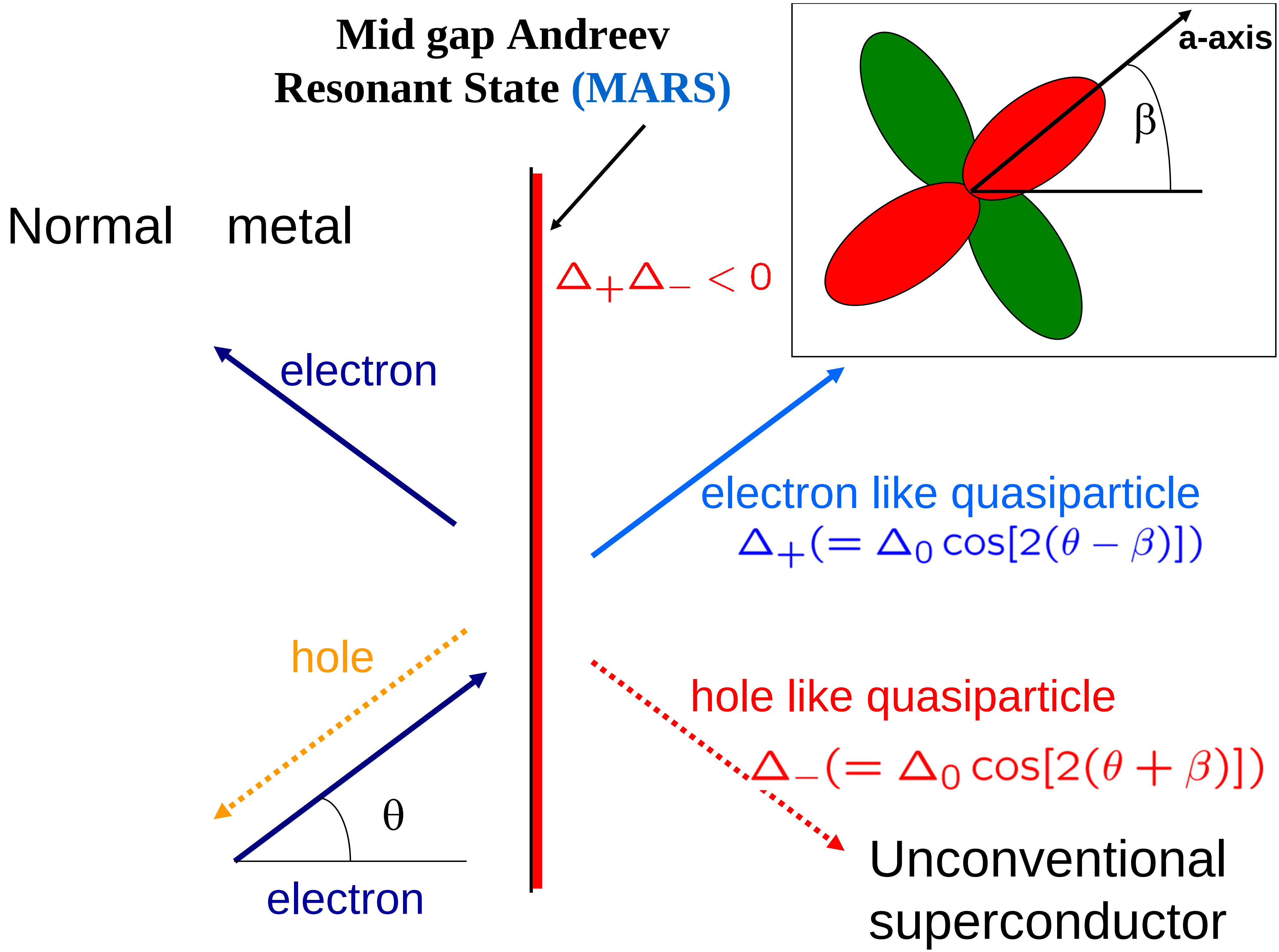


$\Delta(\theta) = \Delta_0 \cos 2\theta$

Singlet unconventional superconductor

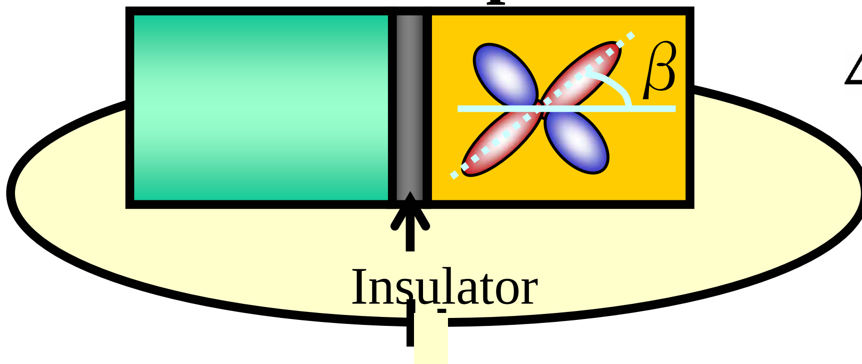
Quasiparticles feel **different signs of the pair potential** depending on direction of their motion

Tunneling spectroscopy has phase sensitivity



Mid gap Andreev
Resonant State (MARS)
a-axis
β
Normal metal
$\Delta_+\Delta_- < 0$
electron
electron like quasiparticle
$\Delta_+(=\Delta_0\cos[2(\theta-\beta)])$
hole
hole like quasiparticle
$\Delta_-(=\Delta_0\cos[2(\theta+\beta)])$
θ
electron
Unconventional
superconductor

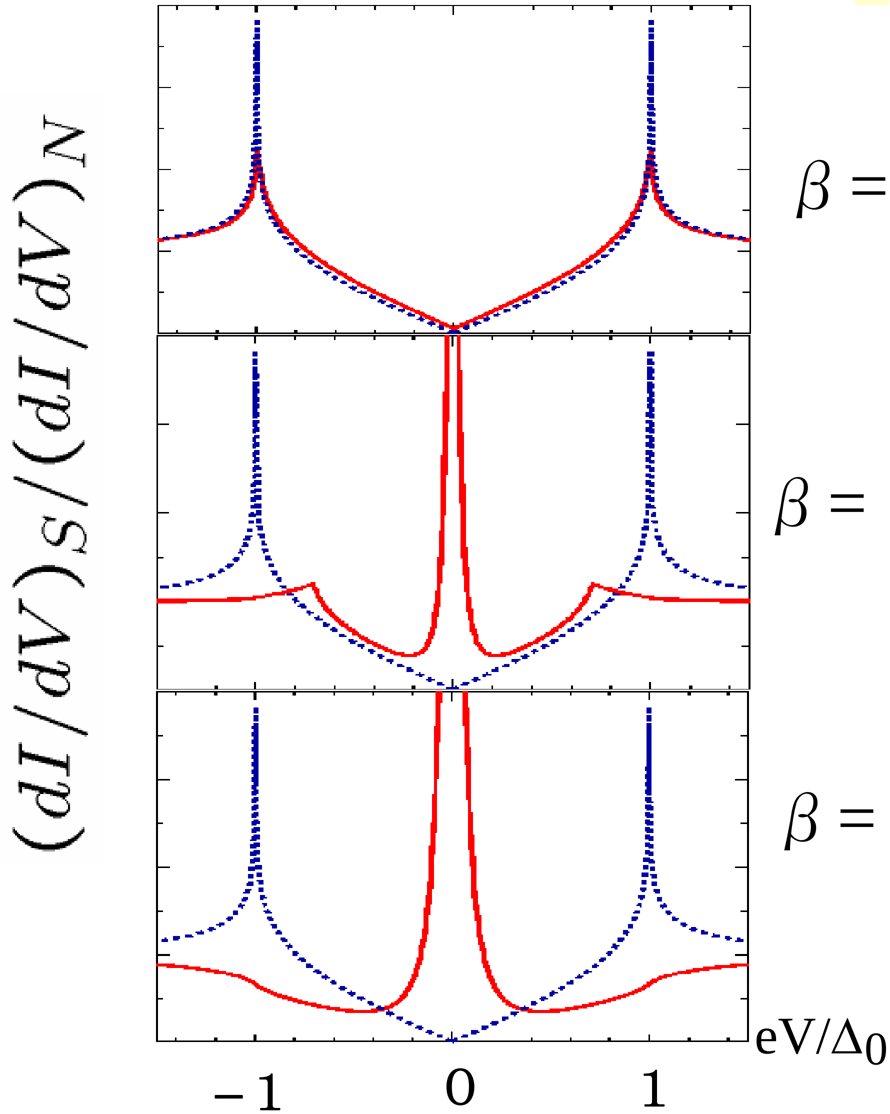
Tunneling conductance in d-wave superconductor junction (ballistic)



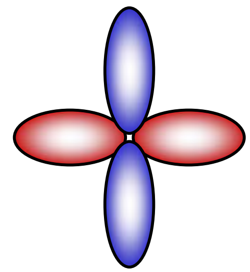
$\Delta_{\pm} = \Delta_0 \cos[2(\theta \mp \beta)]$

Bruder (1990)
Blonder Tinkham Klapwijk (1982)

Blue dotted line：
Bulk d-wave DOS



$\beta = 0$

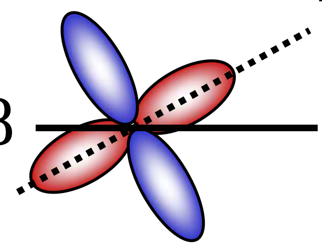


Tanaka Kashiwaya (1995)
Phys Rev Lett 74 3451 (1995)

ZBCP

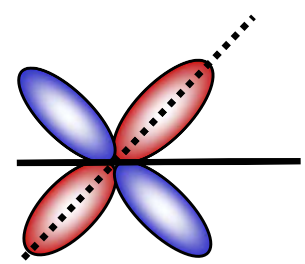
Zero bias conductance peak

$\beta = \pi/8$



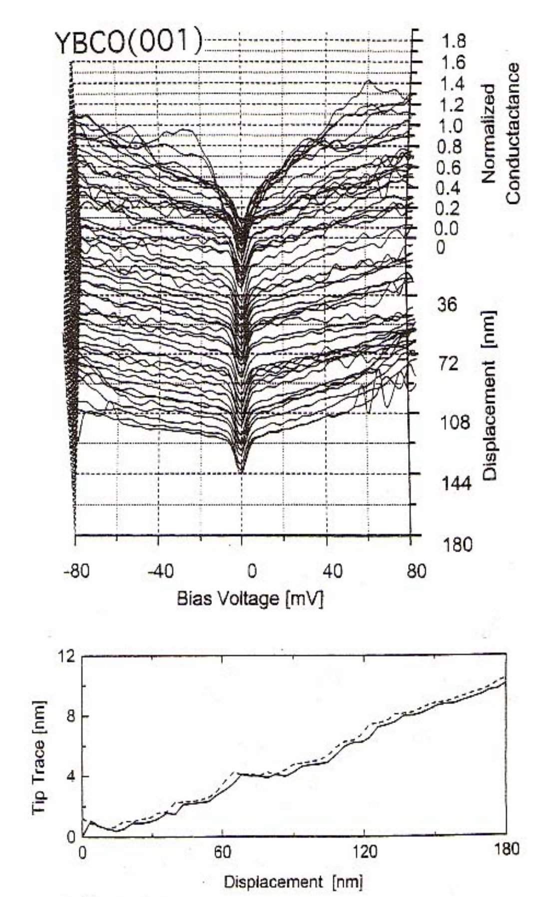
Mid gap Andreev resonant state

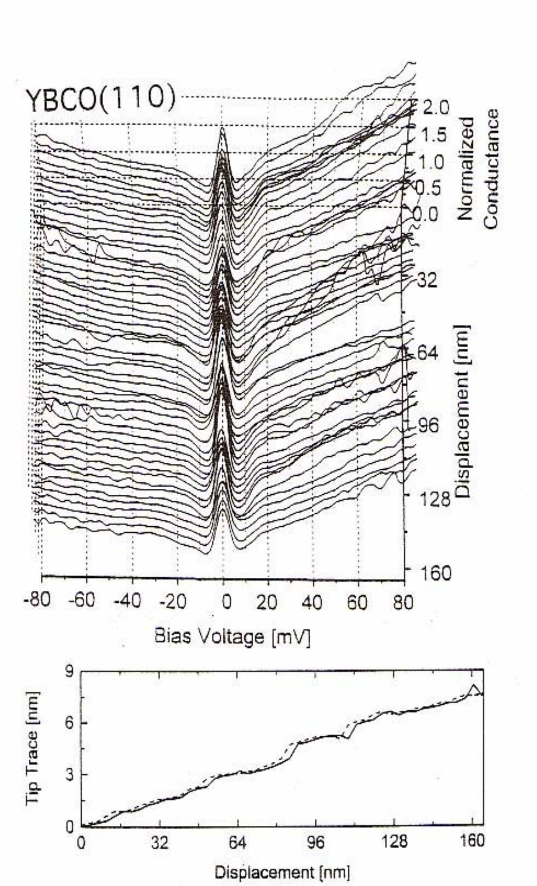
$\beta = \pi/4$

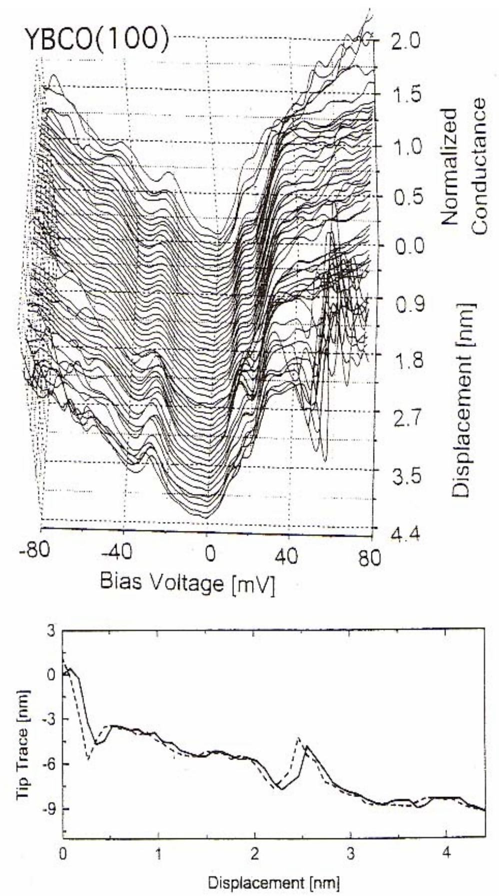


Surface bound state
Hu (1994)
Buchholtz (1981)
Hara Nagai(1986)
Matsumoto Shiba(1995)

Tunneling spectroscopy of YBCO films by STM







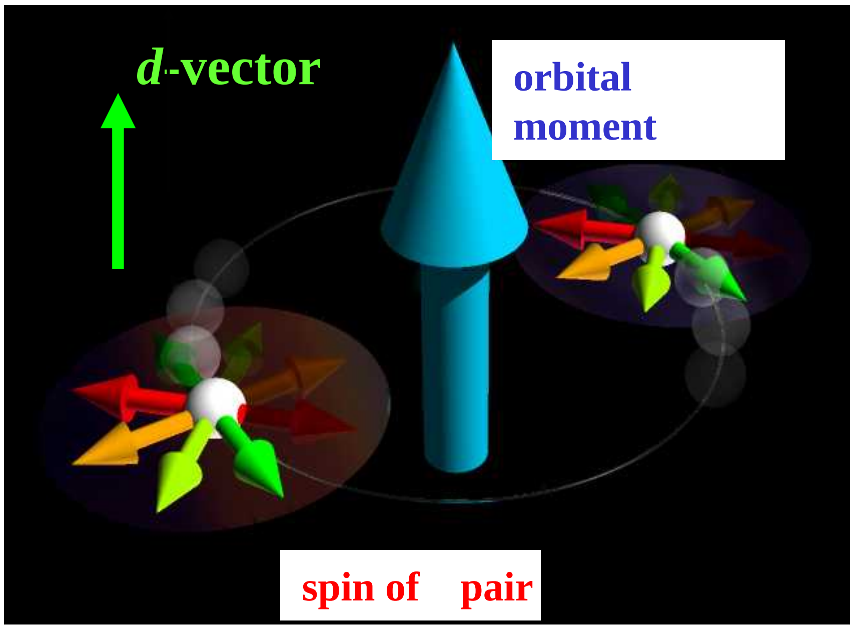
R.H. Greene, Alff, Deutscher, Cucolo, Cheska, Piano, Yeh, Iguchi, R.L. Greene, Koren, Ekin, Lesuer, Geerk, Sato, Oda, Sharoni…

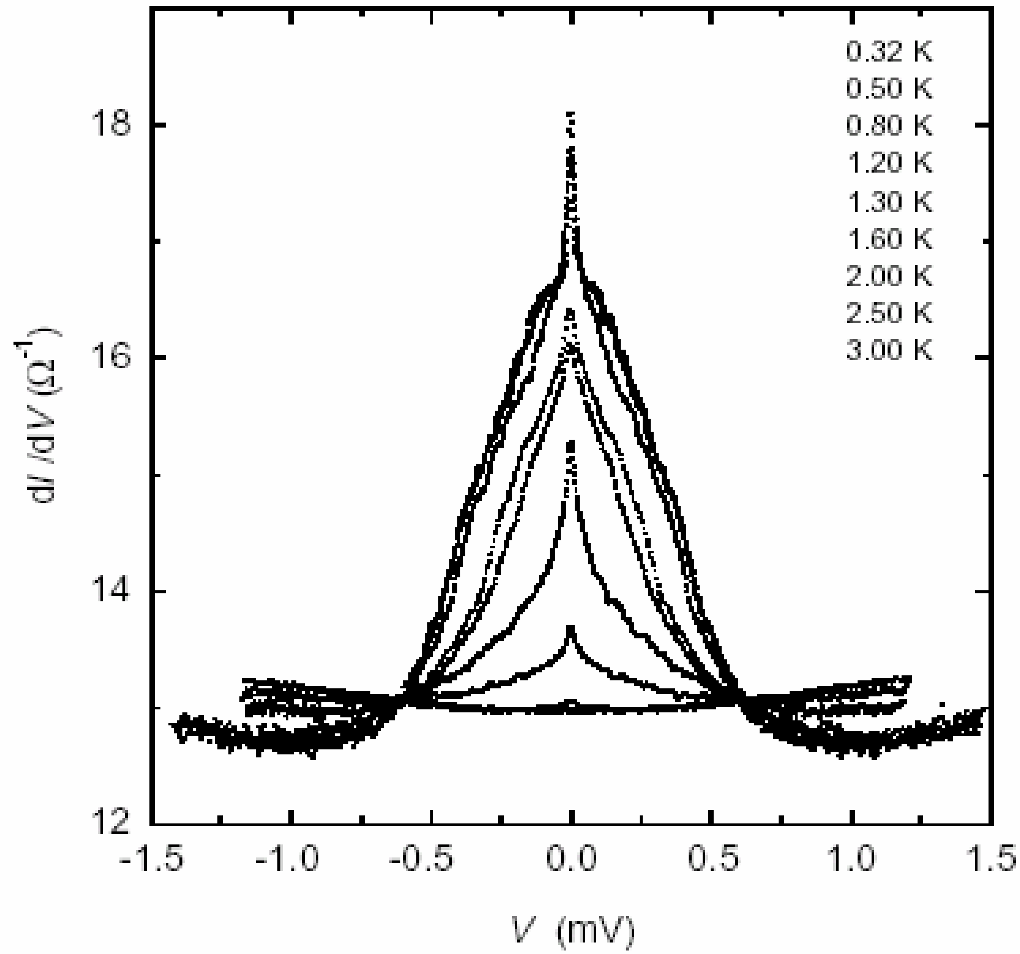
S. Kashiwaya and Y. Tanaka
Rep. Prog. Phys. (2000)

MARS observed in triplet superconductor Sr_2RuO_4

Y. Maeno, G. Bednorz et al.
Nature 372 532 (1994)

(p-wave chiral)

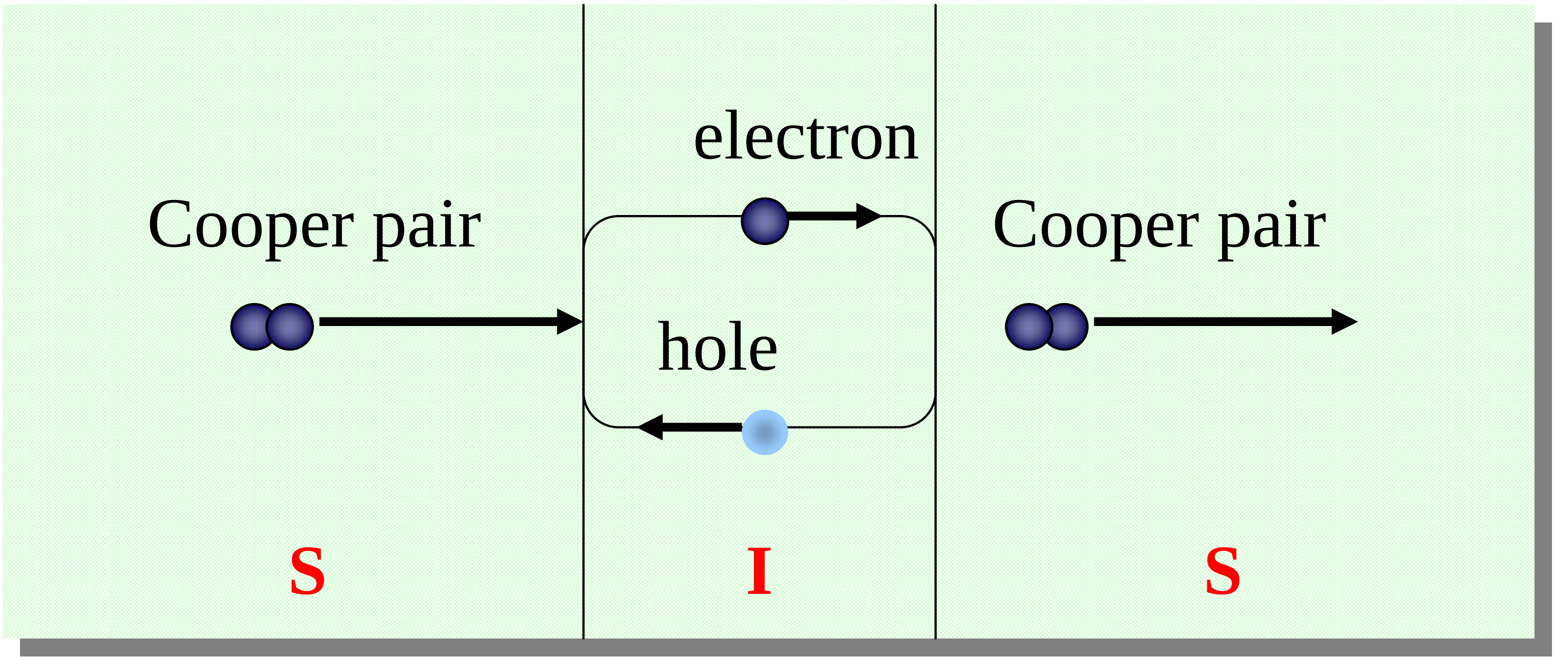




Mao, Nelson, Jin, Liu and Maeno Phys. Rev. Lett. 87, 037003 (2001)

Kawamura, Yaguchi, Kikugawa Maeno Takayanagi
J. Phys. Soc. Jpn. 74 531 (2005)

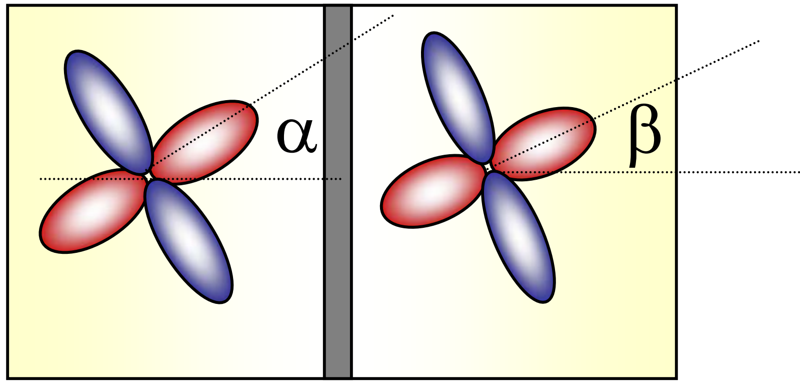
Josephson effect



Josephson current can be expressed in terms of Andreev reflection amplitudes

A. Furusaki and M. Tsukada, Solid State Commun. Vol. 78, 299 (1991).

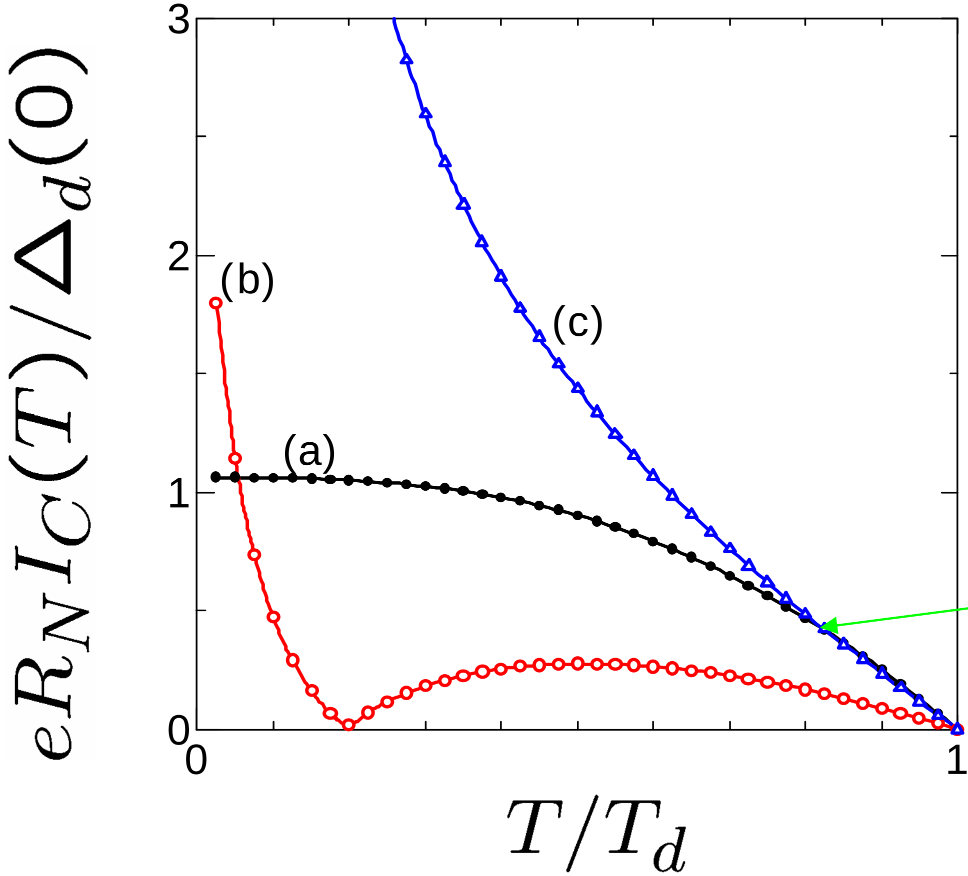
DC Josephson current in d/I/d junctions



$\alpha = -\beta$

Barash (1996)
Tanaka (1996)

Transition from 0 to π junction



(a) $\alpha=0$

(b) $\alpha=0.1\pi$

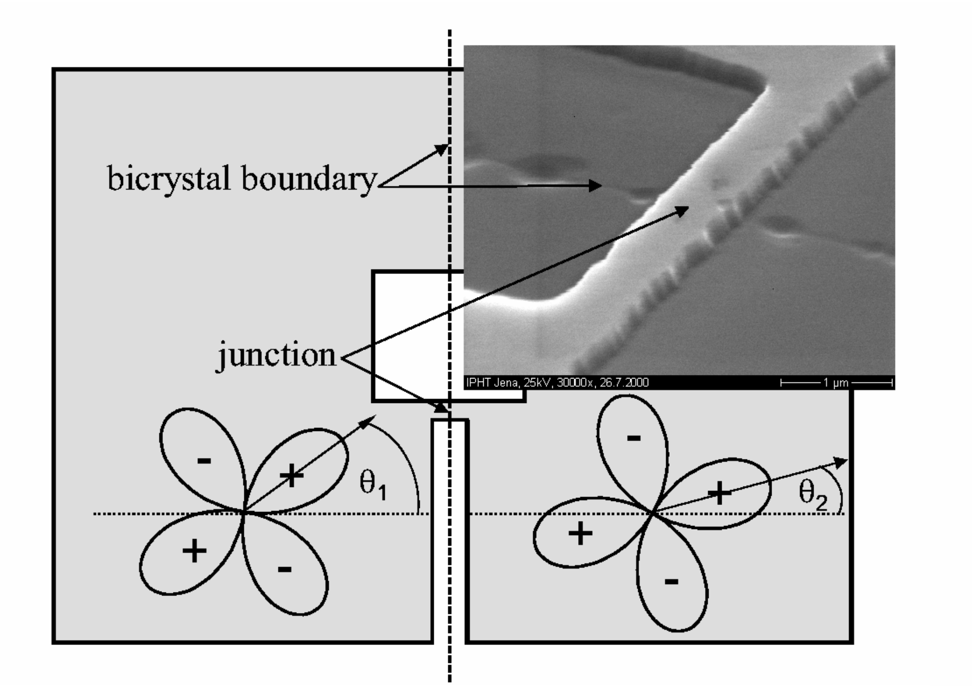
(c) $\alpha=0.25\pi$

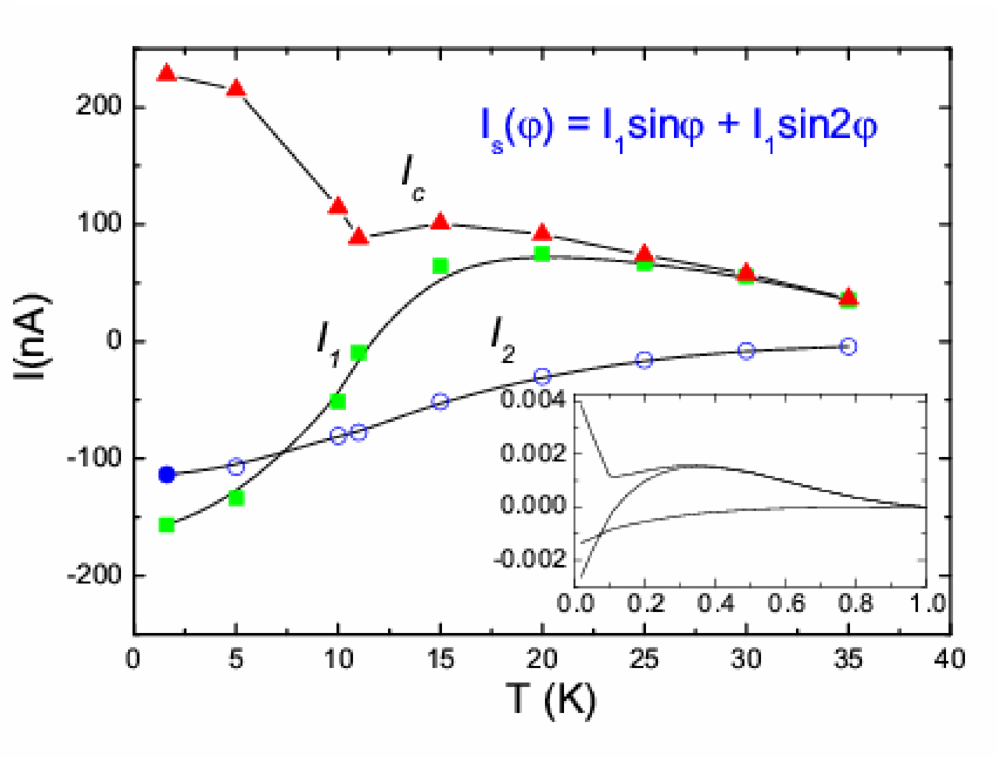
Nonmonotonous temperature dependence

Phys. Rev. B 53 11957(1996)

Phys. Rev. B 56 892(1997)

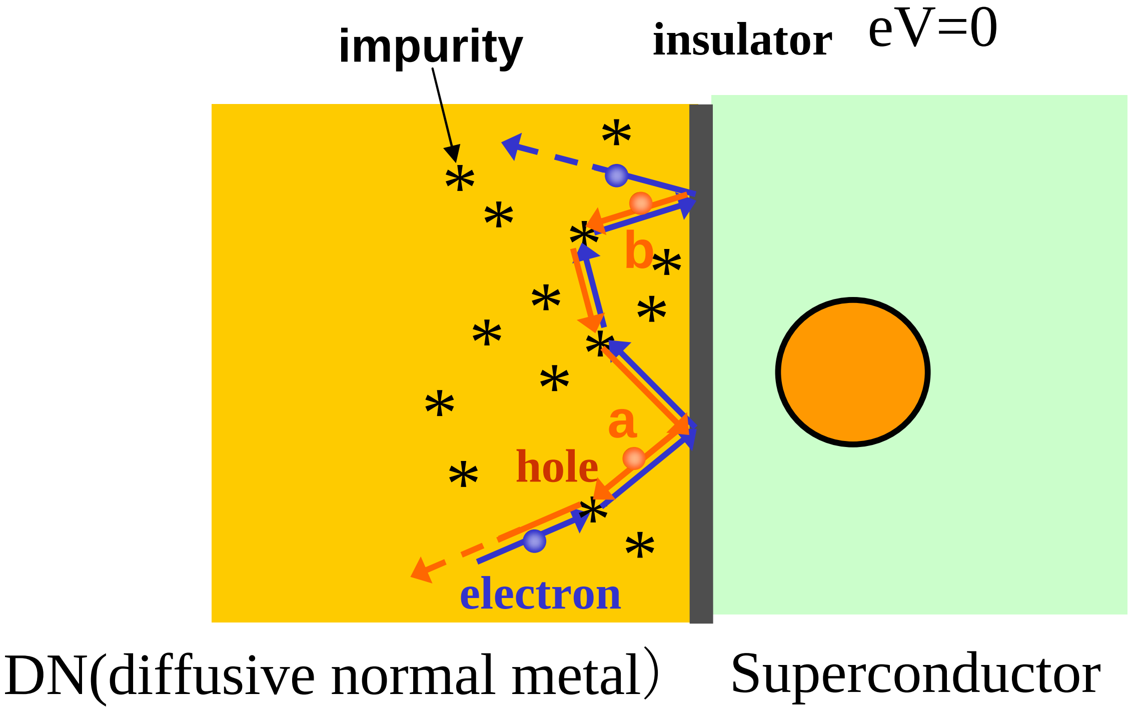
Experiment: E. Il’ichev *et al.*, Phys.Rev.Lett. **86**, 5369 (2001).





$$\alpha_1 = -\alpha_2 = \pi/8$$

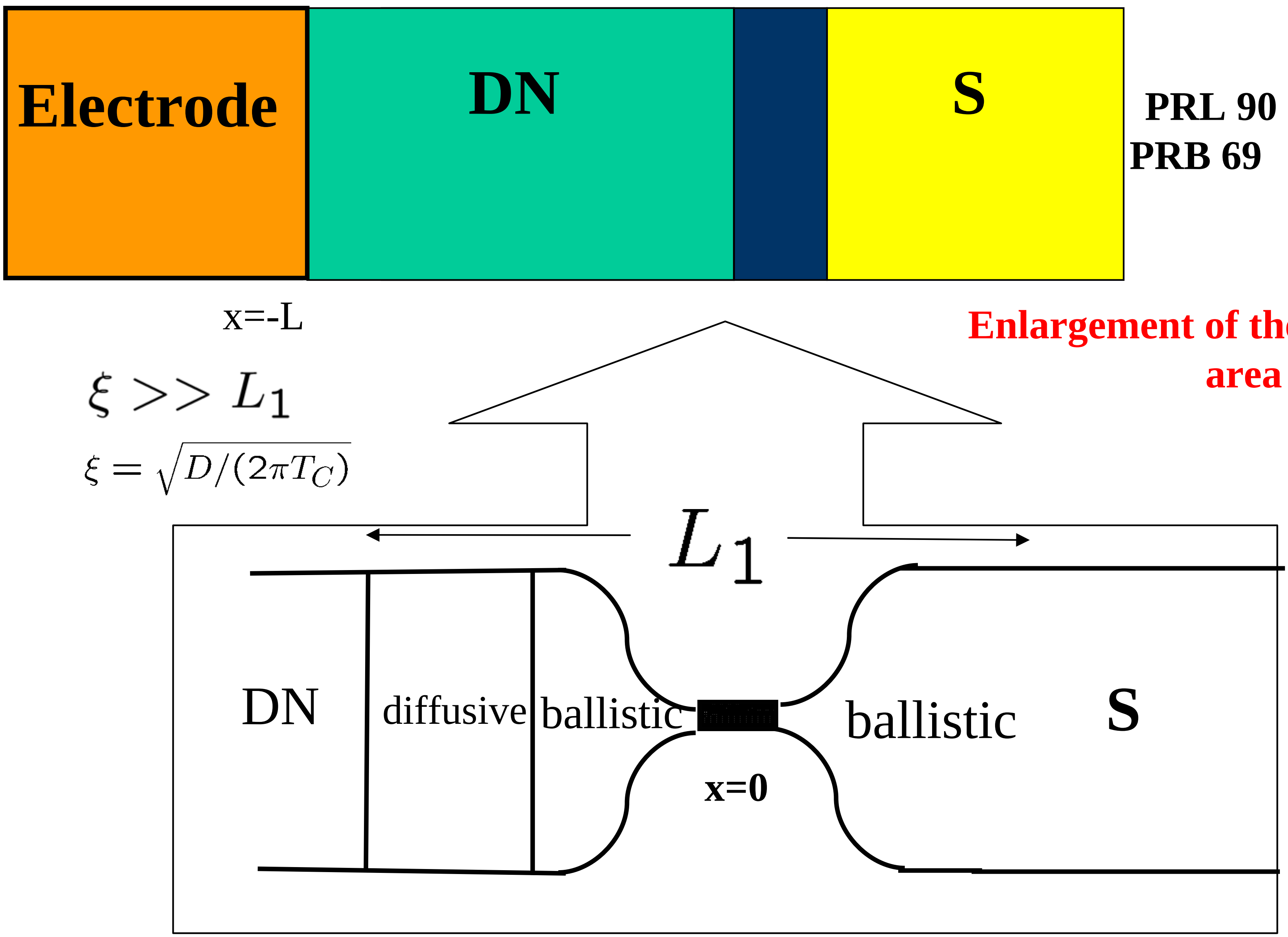
Retro reflectivity of the Andreev reflection in DN



Van Wees *et al*.(1992)

Proximity effect is enhanced by the diffusive scattering

Diffusive normal metal (DN)/ unconventional superconductor (S) junction



PRL 90 167003 (2003)
PRB 69 044519 (2004)

Enlargement of the constriction area

Keldysh Nambu Green's function in DN/S junction

Quasiclassical approximation

Dirty limit in DN (Usadel equation)

clean limit in S (Asymptotic Green's function is given)

Usadel equation (Green's function in DN)

$$D\frac{\partial}{\partial x}[\check{G}_N(x)\frac{\partial \check{G}_N(x)}{\partial x}] + i[\check{H}, \check{G}_N(x)] = 0,$$

$$\check{H} = \begin{pmatrix} \hat{H}_0 & 0 \\ 0 & \hat{H}_0 \end{pmatrix}, \quad \hat{H}_0 = \epsilon\tau_3.$$

D Diffusion constant

$\check{G}_N(x)$ is angular averaged Green's function

$\check{G}_N(x)$ is detemined by solving Usadel equation based on **a new boundary condition**

Electric current

$$I_{el} = \frac{-L}{4eR_D}\int_0^\infty d\epsilon \mathrm{Tr}[\hat{\tau}_3(\check{G}_N(x)\frac{\partial \check{G}_N(x)}{\partial x})^K],$$

Boundary condition at DN/S interface (Y.Nazarov)

$$\frac{L}{R_D}\left(\check{G}_N \frac{\partial \check{G}_N}{\partial x}\right)_{|x=0_-} = (2e^2 R_B)^{-1} < -h\check{I} >,$$

$$< \check{I} >= \int_{-\pi/2}^{\pi/2} \check{I} \cos\phi d\phi / \int_{-\pi/2}^{\pi/2} T(\phi) \cos\phi d\phi$$

Balance equation expressed by Keldysh-Nambu Green's function

R_D resistance in DN

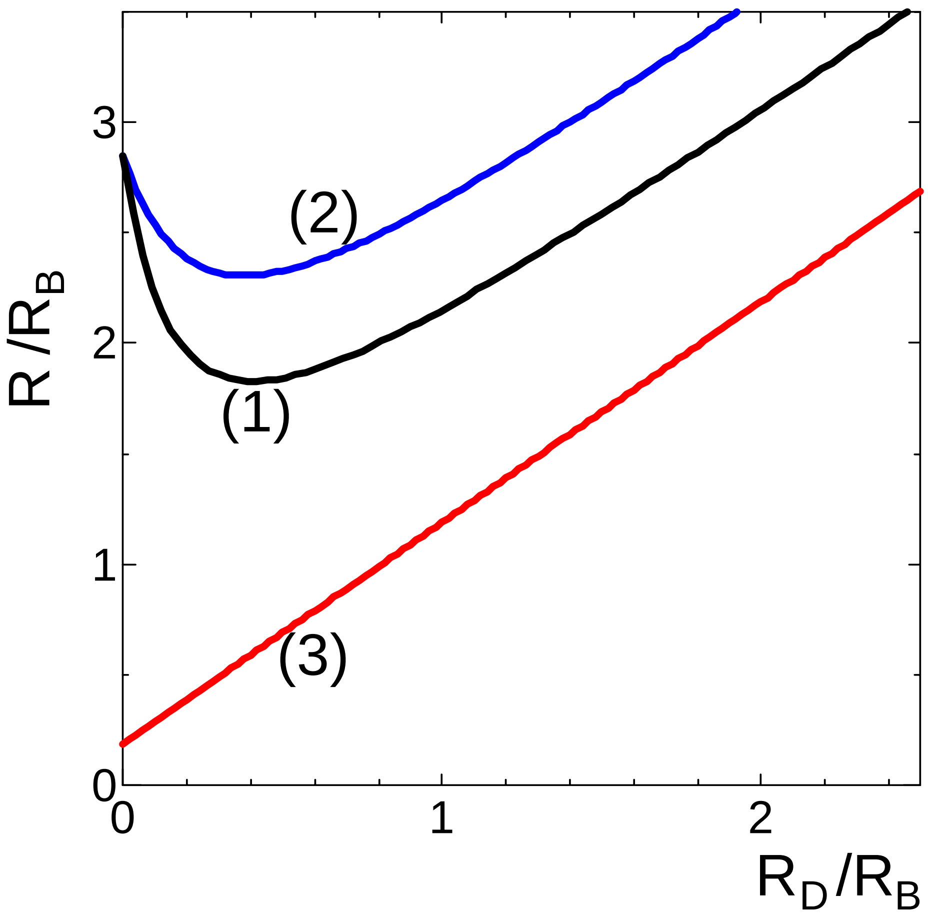
R_B resistance at the interface

$R_B = R_0$ for $T(\phi) = 1$

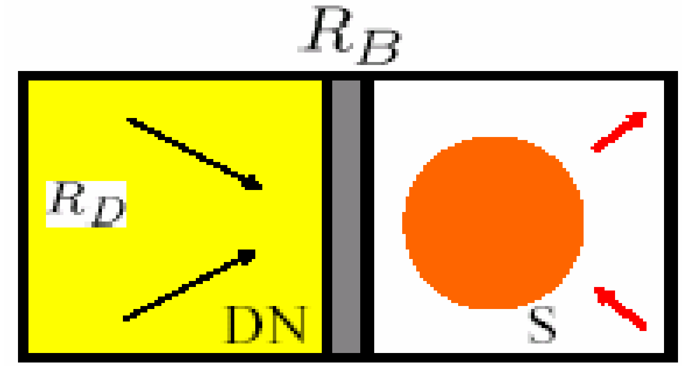
General expression of the matrix current $\check{I}$

PRL 90 167003 (2003)

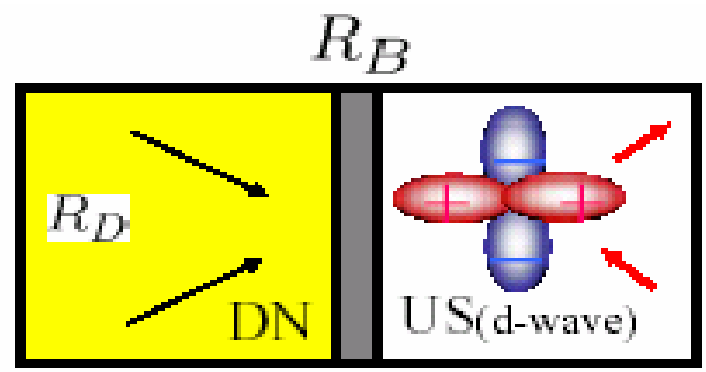
Zero voltage total resistance



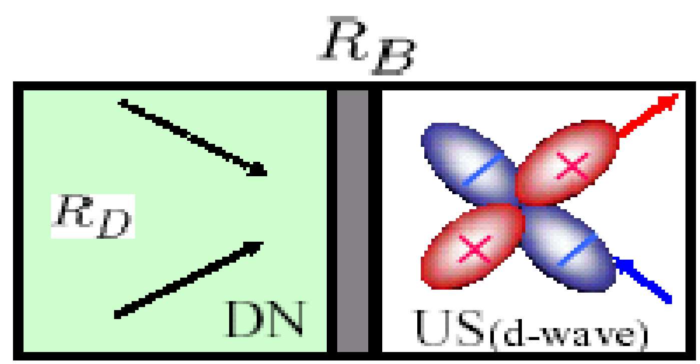
(1



(2



(3)



$\theta(x) = 0$

(1) Proximity no MARS (reentrance)

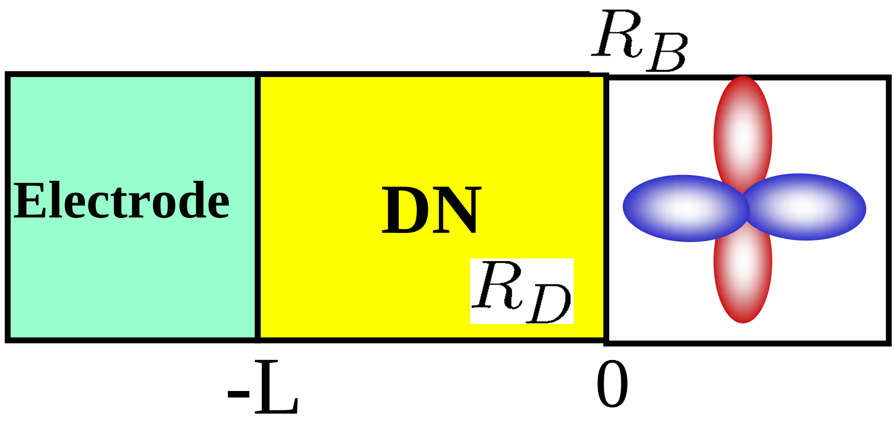
(2) Proximity no MARS (reentrance)

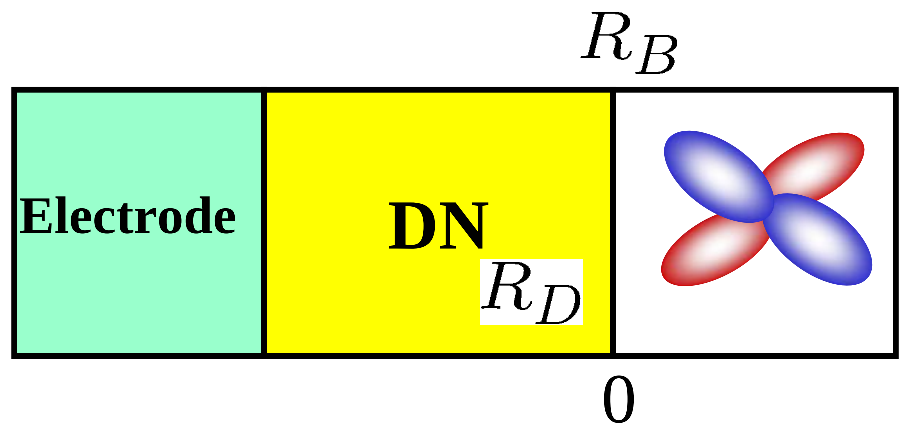
(3) No Proximity MARS (Ohm's rule)

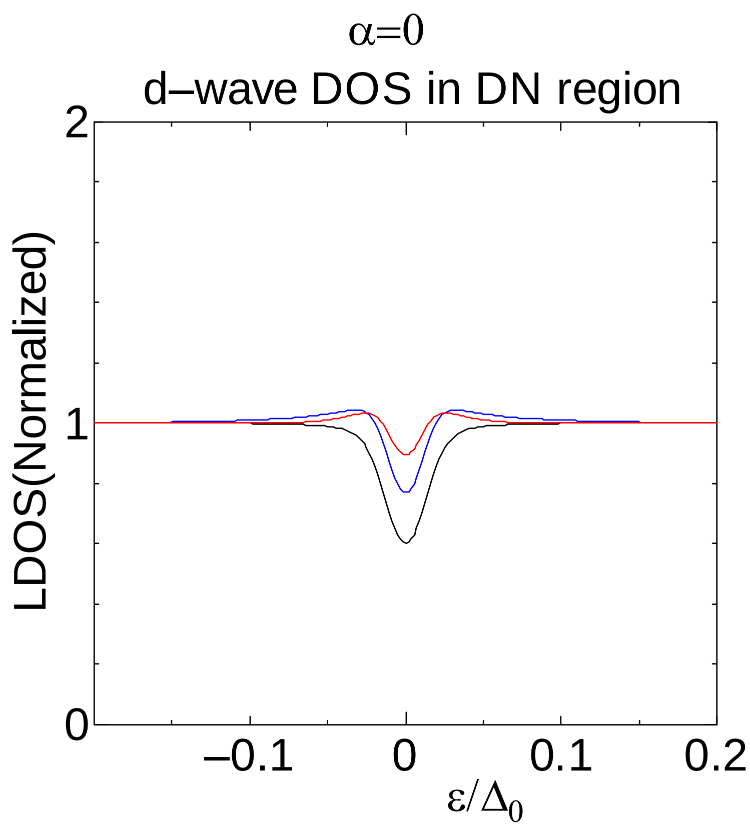
R_0; Sharvin resistance

$$R_B = 2R_0 / \int_{-\pi/2}^{\pi/2} T(\phi) \cos\phi d\phi$$

Local density of states in DN (d-wave)







Z=10

$$L/\xi = 18$$

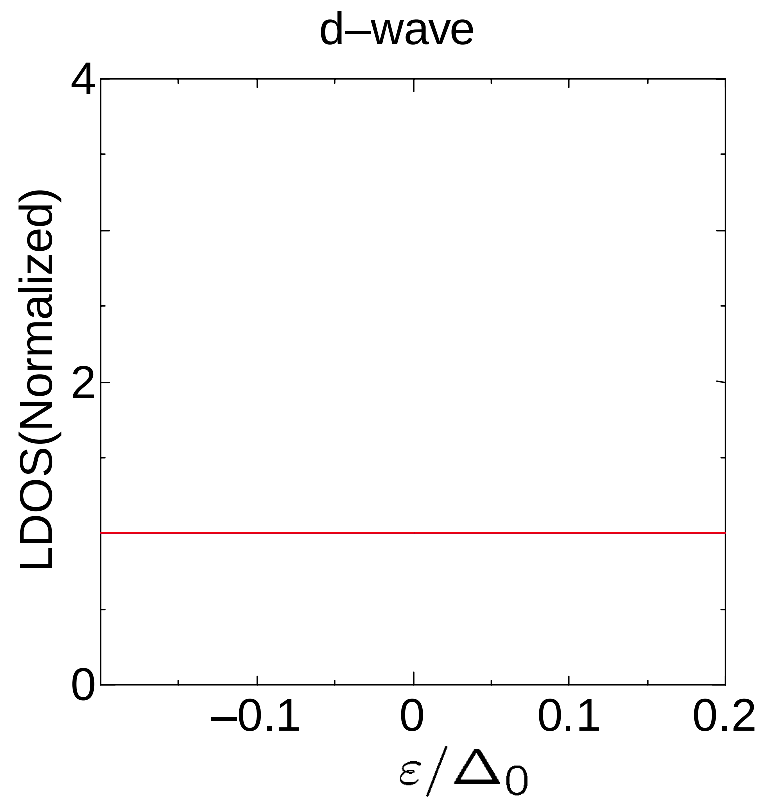
$$E_{th}/\Delta_0 = 0.01$$

$$\xi = \sqrt{D/(2\pi T_C)}$$

x=0

x= –L/4

x= –L/2



Energy gap due to proximity effect

No proximity effect

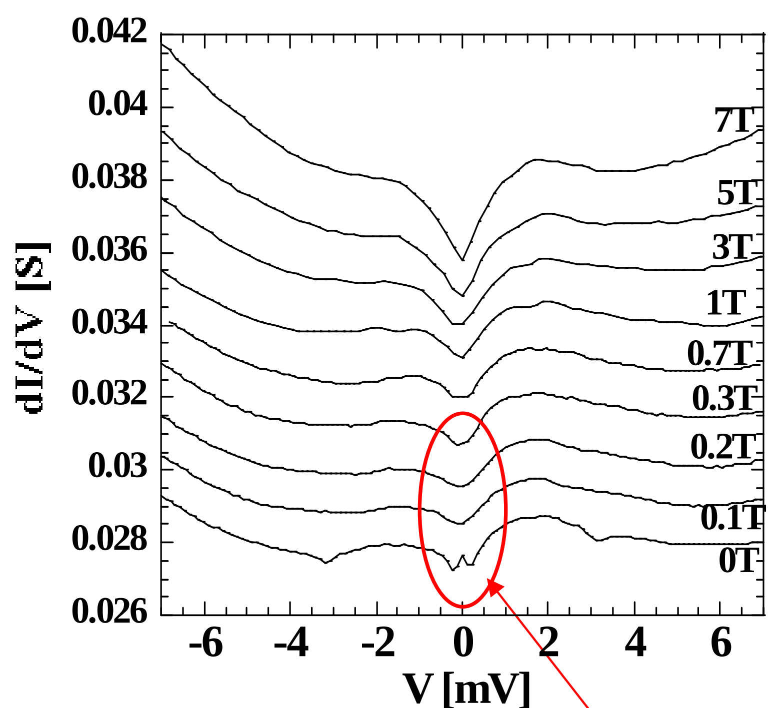
Proximity effect in d-wave junction
ZBCP by proximity effect not by MARS

Experiments : size effect (<10μm) of YBCO junctions

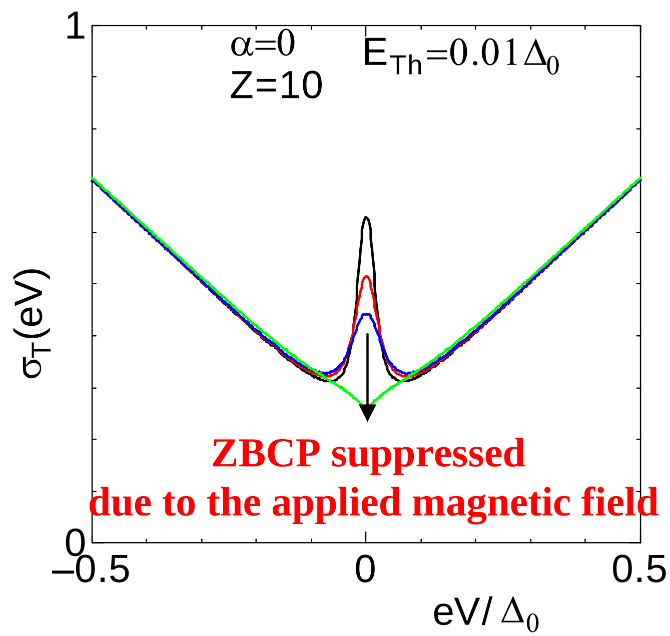
H. Kashiwaya, PRB 68, 054527

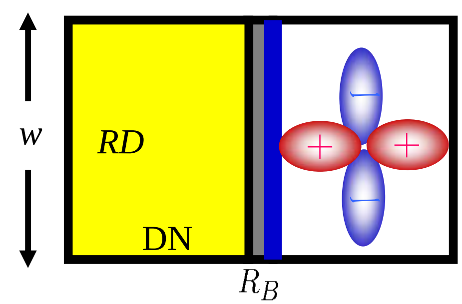
PRL 90 167003 (2003)

without MARS



consistent



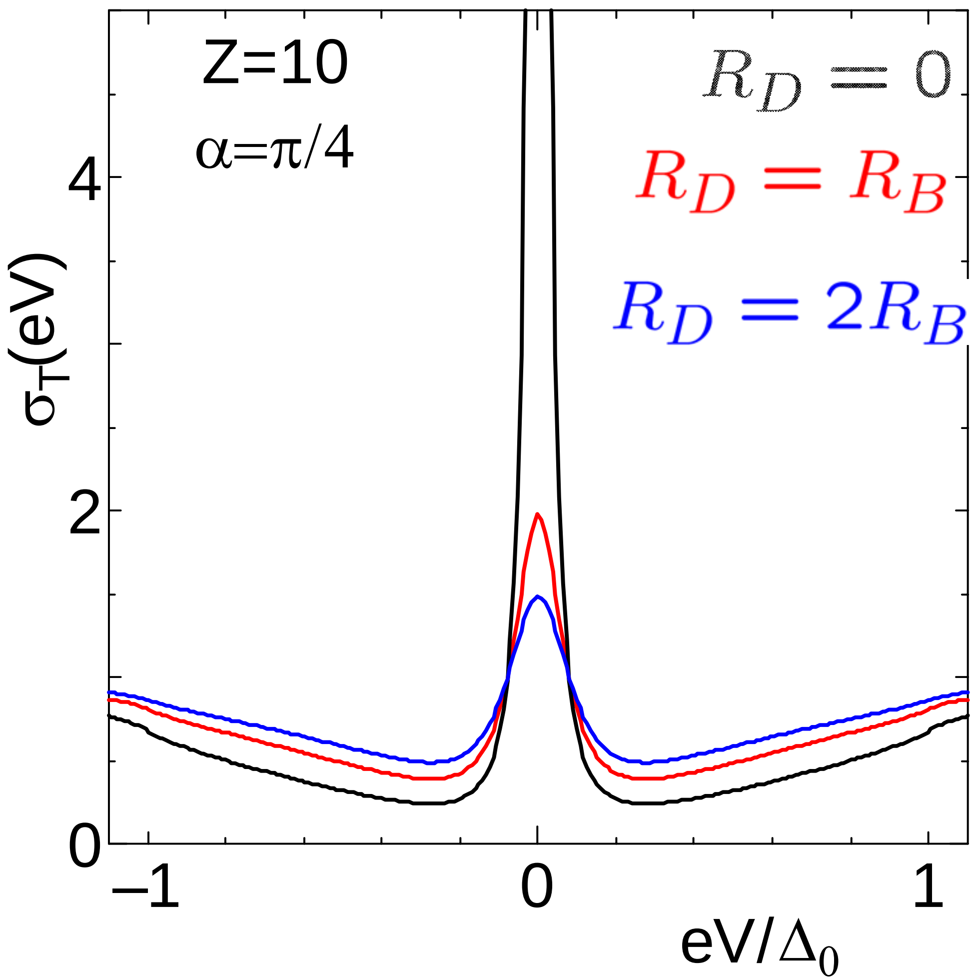


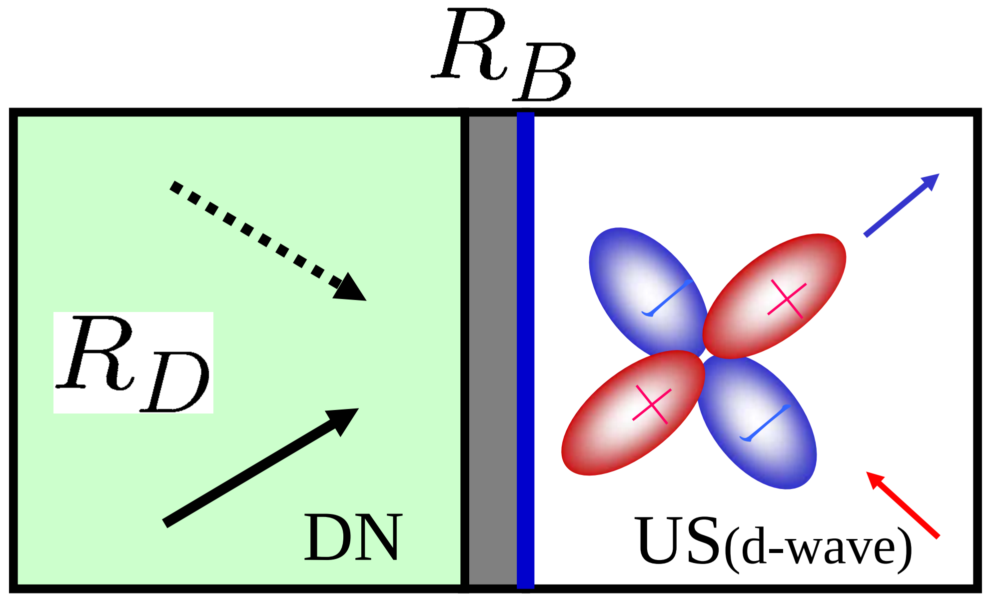
Peak disappears by small applied field due to decoherence

Careful treatment of experimental data is required to test the symmetry due to the appearance of ZBCP. Magnetic field response can be used to identify the origin.

ZBCP by MARS & diffusive scattering

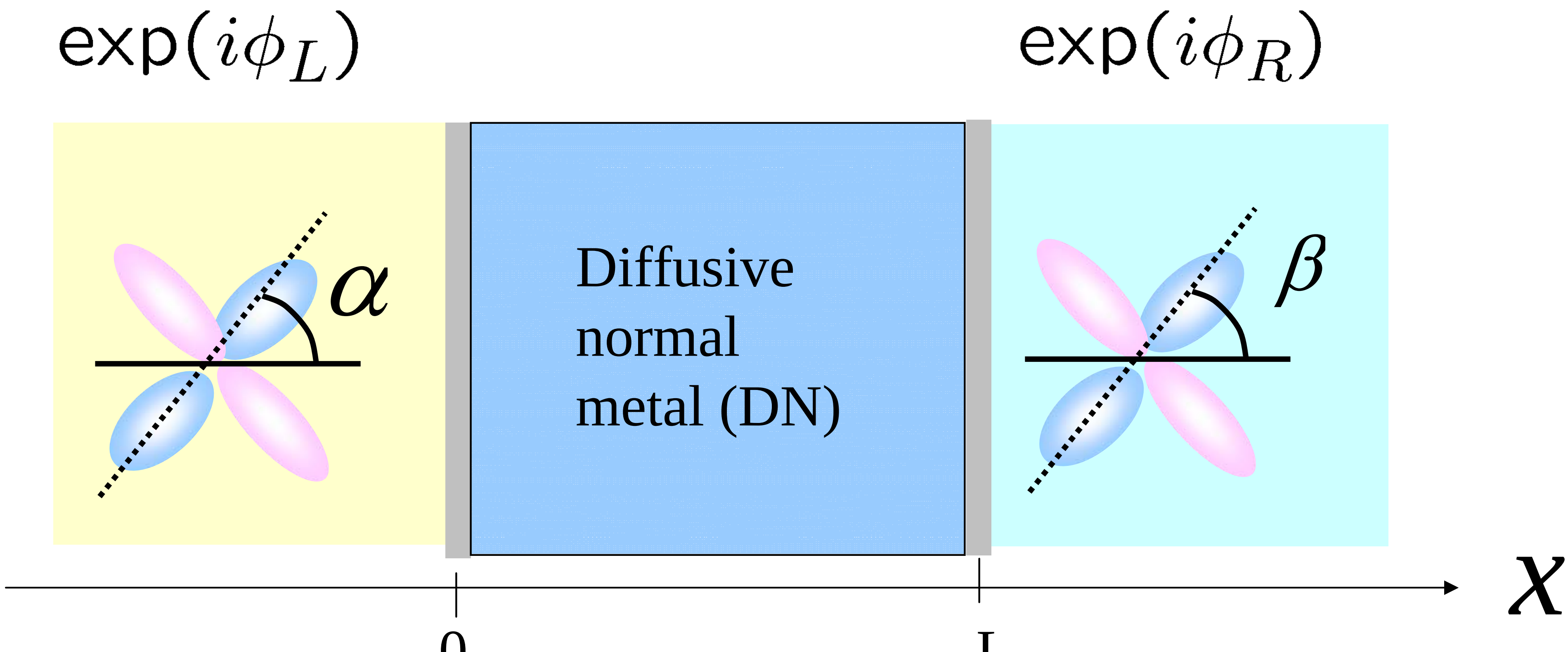
No proximity effect in DN





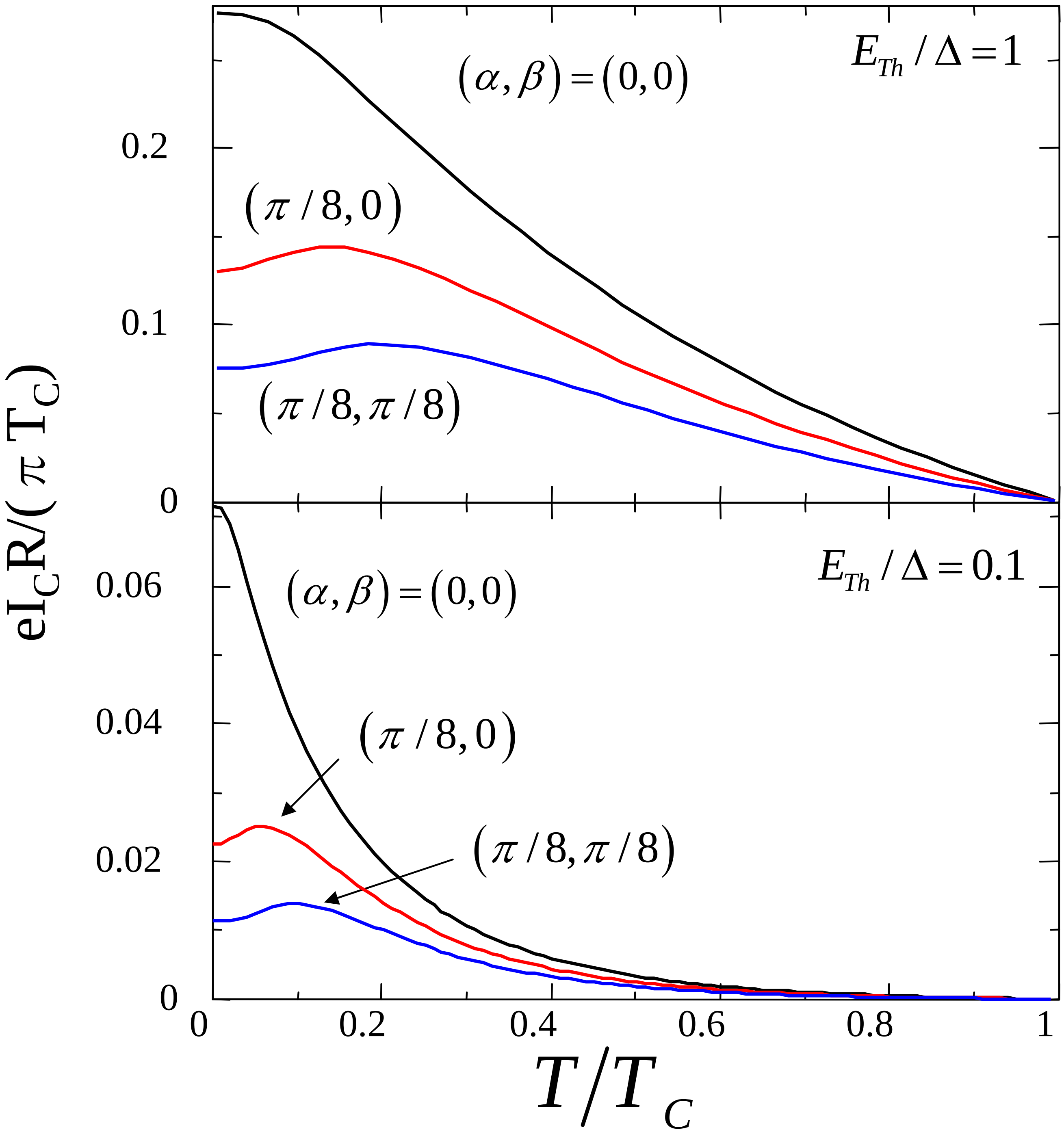
Height of the ZBCP is reduced by R_D

Josephson effect in d-wave / DN/ d-wave junction

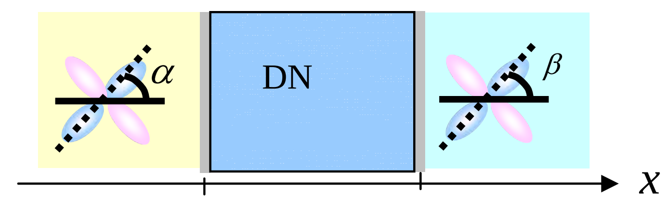


$$\phi = \phi_L - \phi_R$$

Nonmonotonic T-dependence of critical current due to competition between MARS and proximity effect.



$$E_{Th}=\frac{D}{L^2}$$

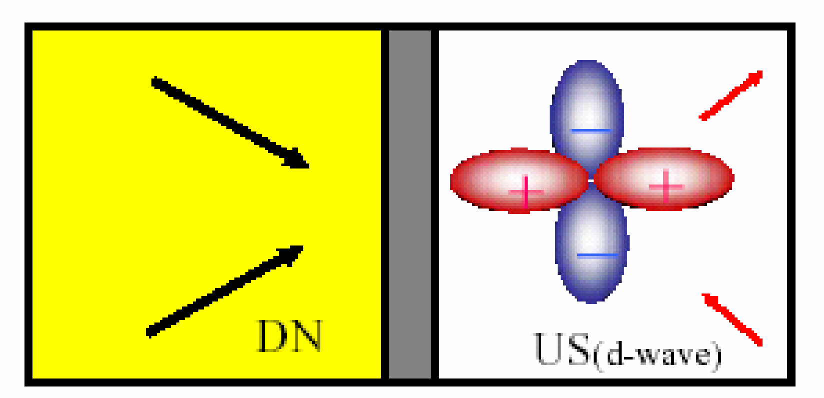


The position of peak shifts to the low T with the decrease of E_{th}.

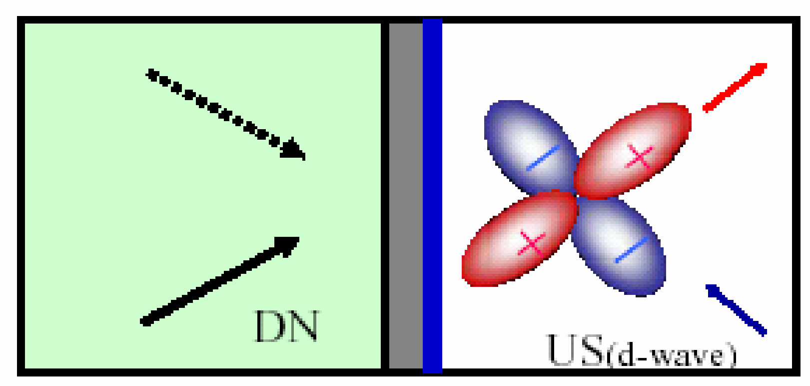
DN/d-wave junctions

PRL 90 167003 (2003)
PRB 69 044519 (2004)

The competition between proximity effect and MARS

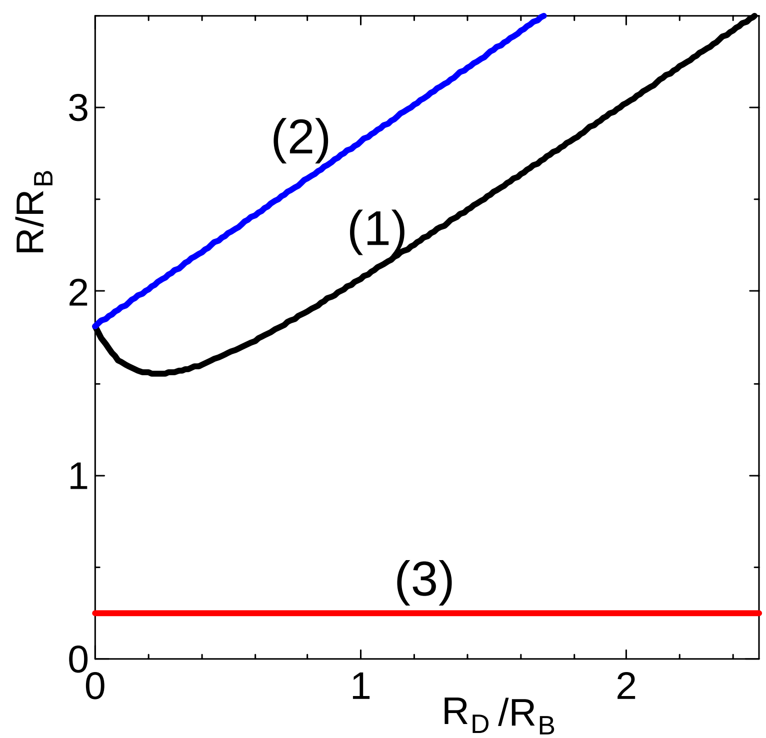


Proximity effect ○
MARS ×



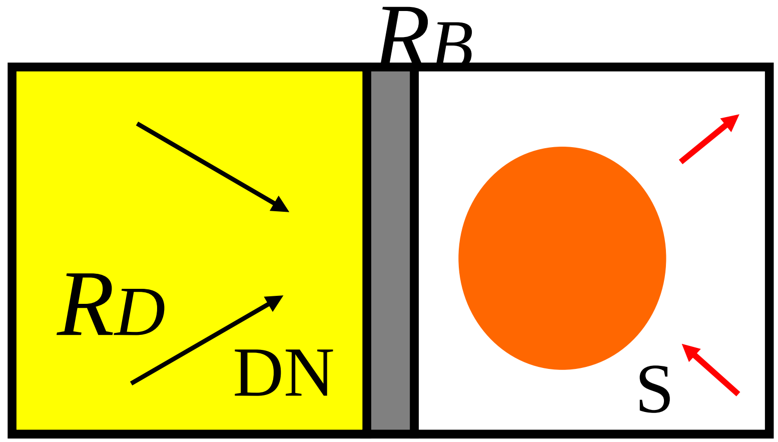
Proximity effect ×
MARS ○

Triplet junctions: resistance

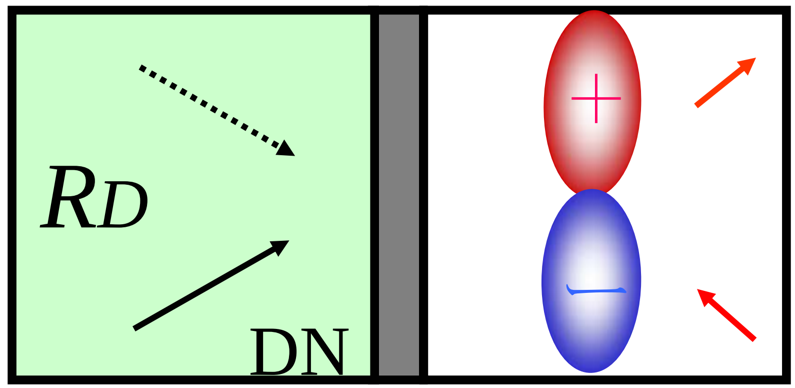


R_D Resistance in DN
R_B Resistance at the interface

(1)

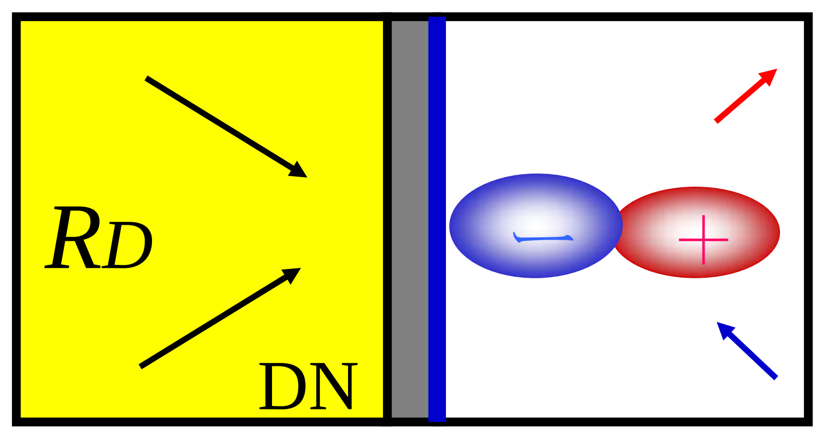


(2)



(no proximity& no MARS)

(3)



(proximity & MARS)

(2) p_y-wave $\quad R = R_D + R_{R_D=0}$

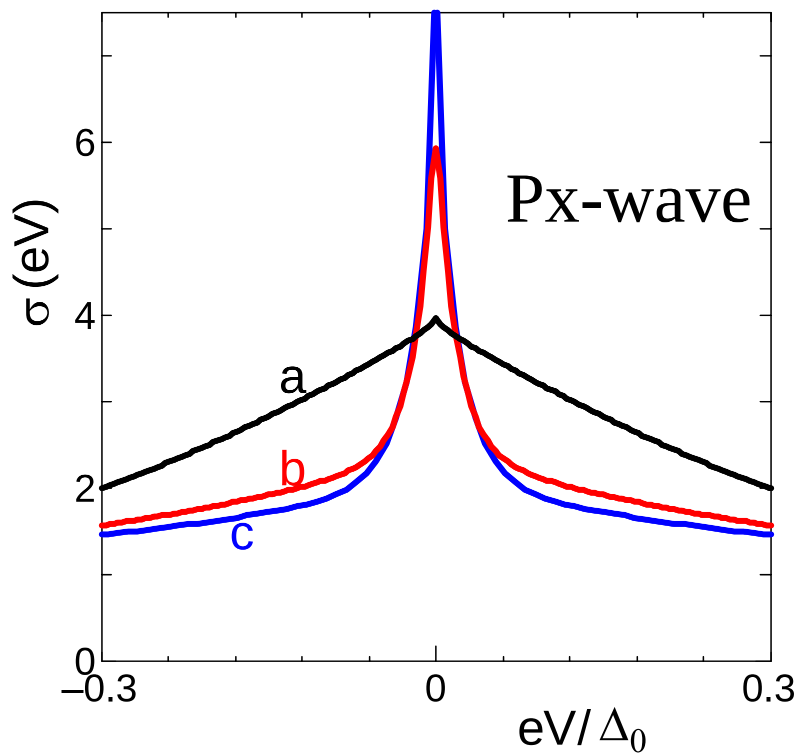
(3) p_x-wave

R is independent of R_D!!

$$R = R_0/2$$

$R_B = 2R_0 / \int_{-\pi/2}^{\pi/2} T(\phi)\cos\phi\, d\phi$ $\quad R_0$; Sharvin resistance

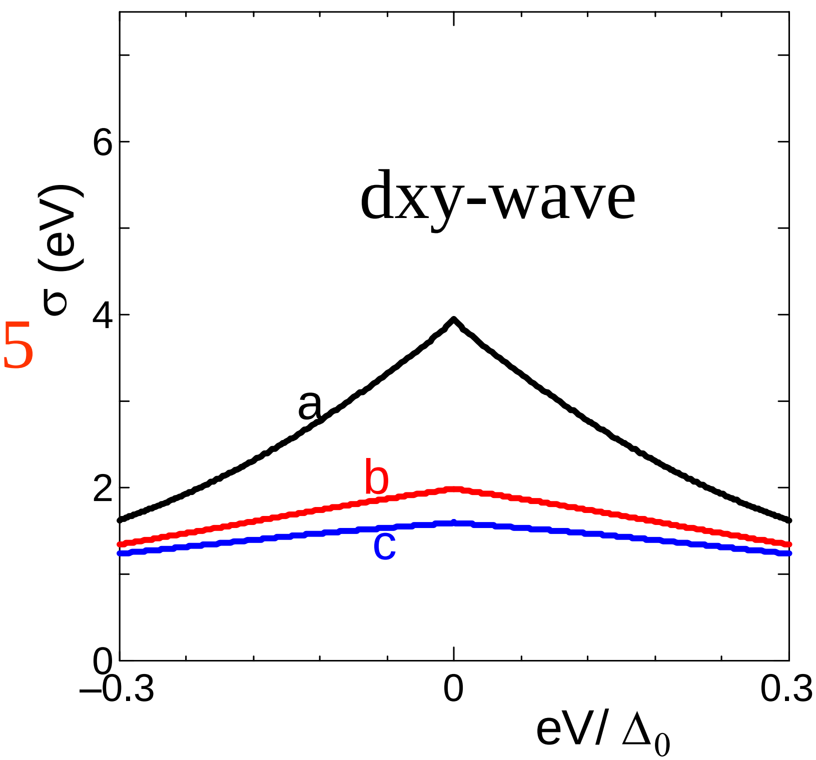
Normalized tunneling conductance

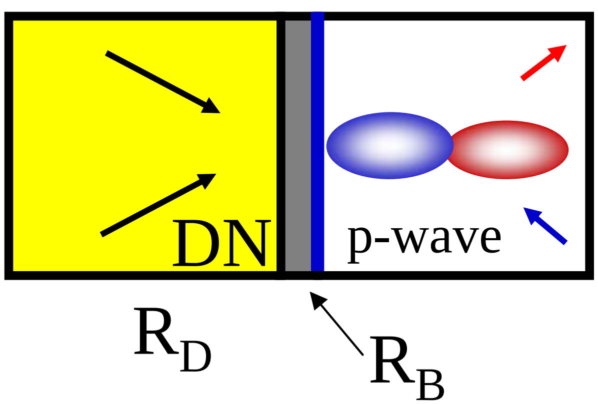


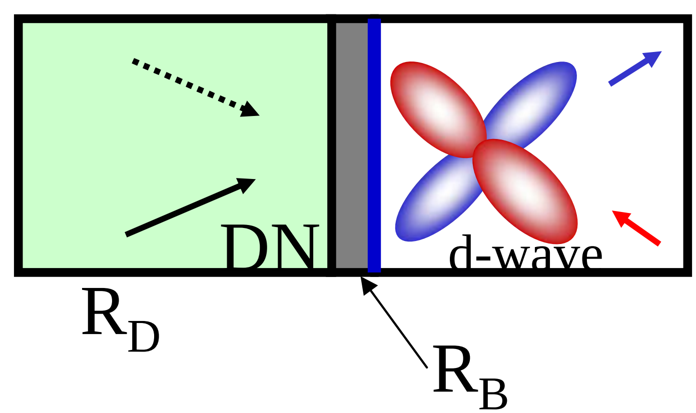
$R_D/R_B=0$

$R_D/R_B=0.5$

$R_D/R_B=1$







ZBCP : MARS R_D/R_B is small

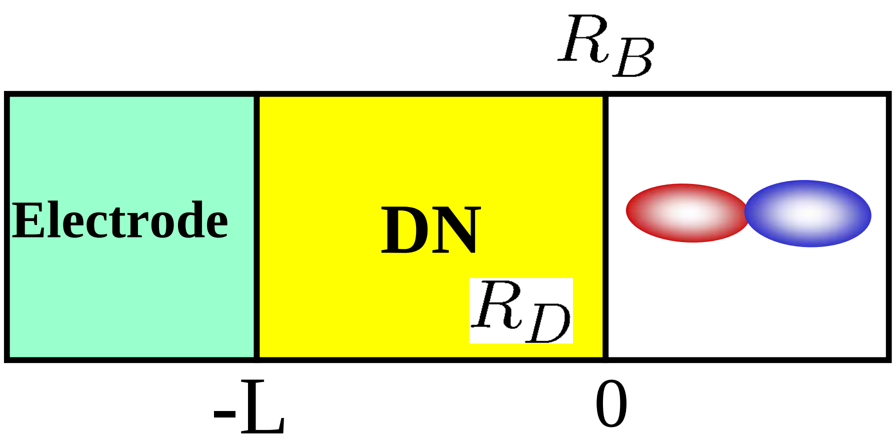
ZBCP : proximity effect

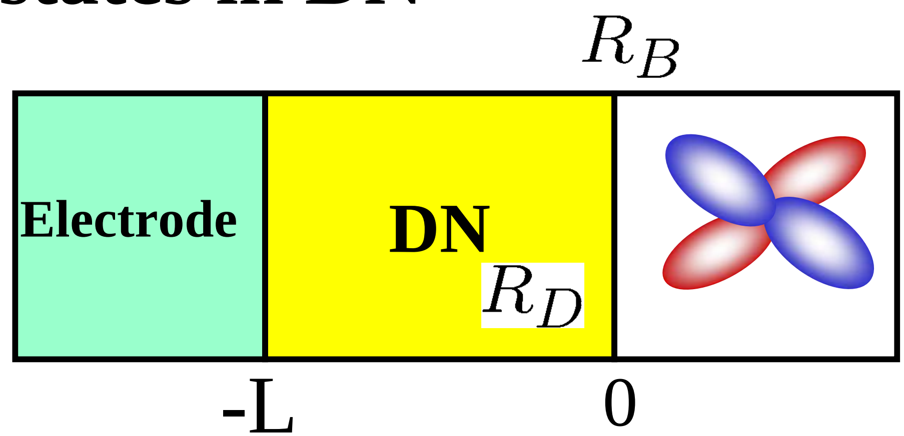
R_D/R_B is large **Giant ZBCP**

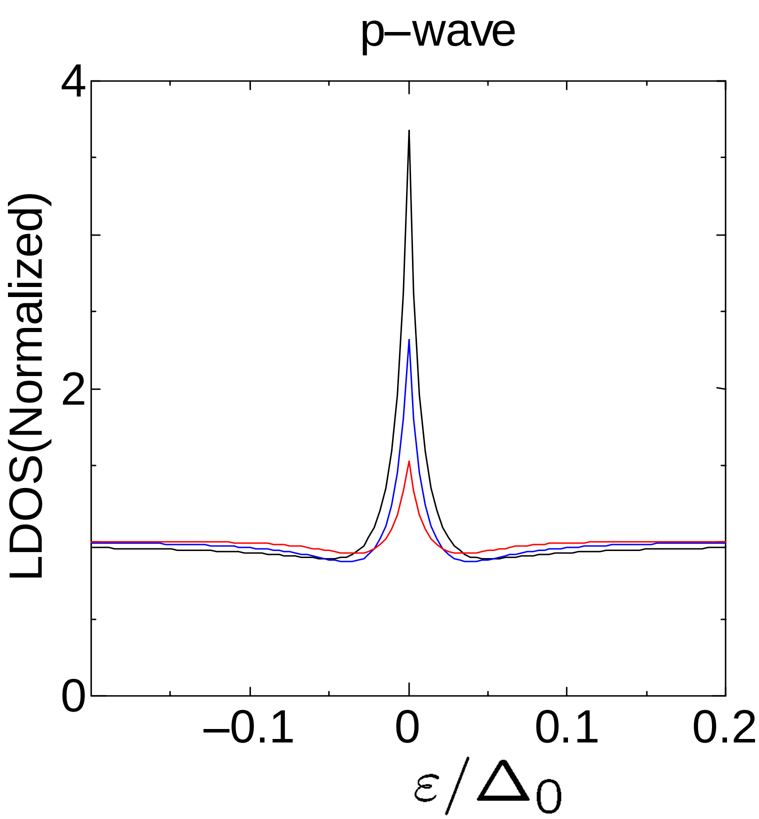
ZBCP only due to MARS

No proximity effect

Local density of states in DN







Z=1.5

$R_D/R_B = 0.5$

$E_{th}/\Delta_0 = 0.02$

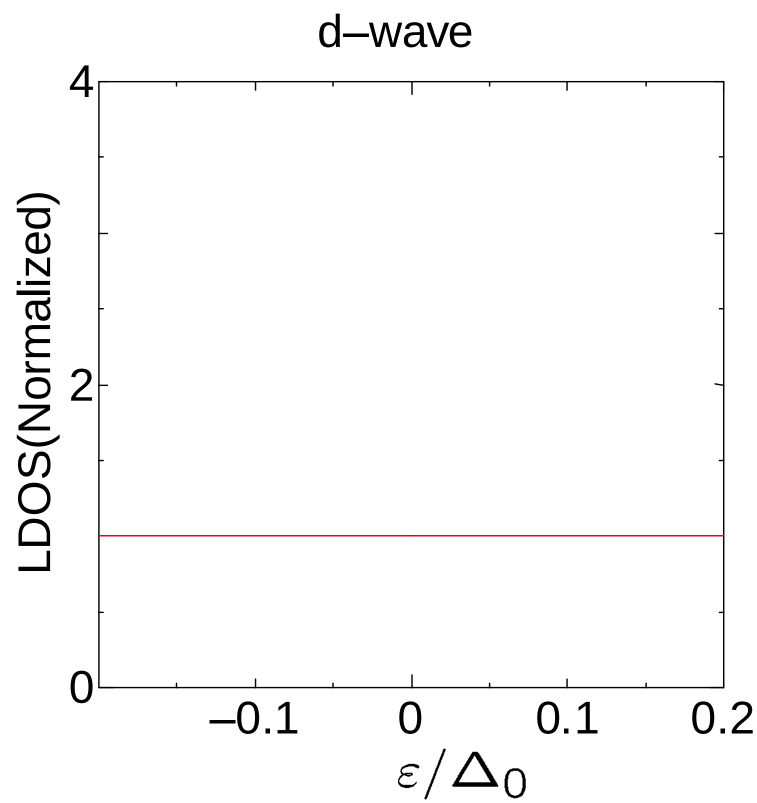
$L/\xi = 13$

$\xi = \sqrt{D/(2\pi T_C)}$

x=0

x=–L/4

x=–L/2



Zero energy peak (ZEP) occurs only in triplet junctions

LDOS at $\epsilon = 0$ $\rho(x)$ $\qquad \rho(x) = \cosh[\frac{2R_D(x+L)}{LR_0}]$

The origin for the DoS anomaly in the triplet case: Unusual energy dependence of pair amplitude

Singlet junctions

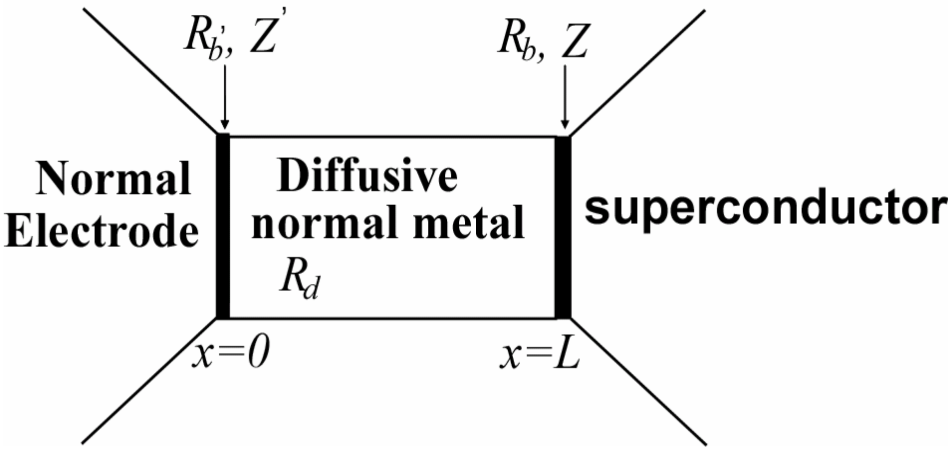
$$f_N(\varepsilon) = f_N^*(-\varepsilon)$$

Triplet junctions

$$f_N(\varepsilon) = -f_N^*(-\varepsilon)$$

This is different from the odd-frequency triplet pairing in SF hybrids (Bergeret, Volkov, Efetov)

Meissner effect



$$\hat{R}_N(x) = \sin\theta\hat{\tau}_2 + \cos\theta\hat{\tau}_3$$

$$j(x) = \pi e^2 N(0) D T \sum_{\omega_n} \mathrm{Trace}[\hat{\tau}_3 \hat{R}_N(x)[\hat{\tau}_3, \hat{R}_N(x)]] A(x)$$

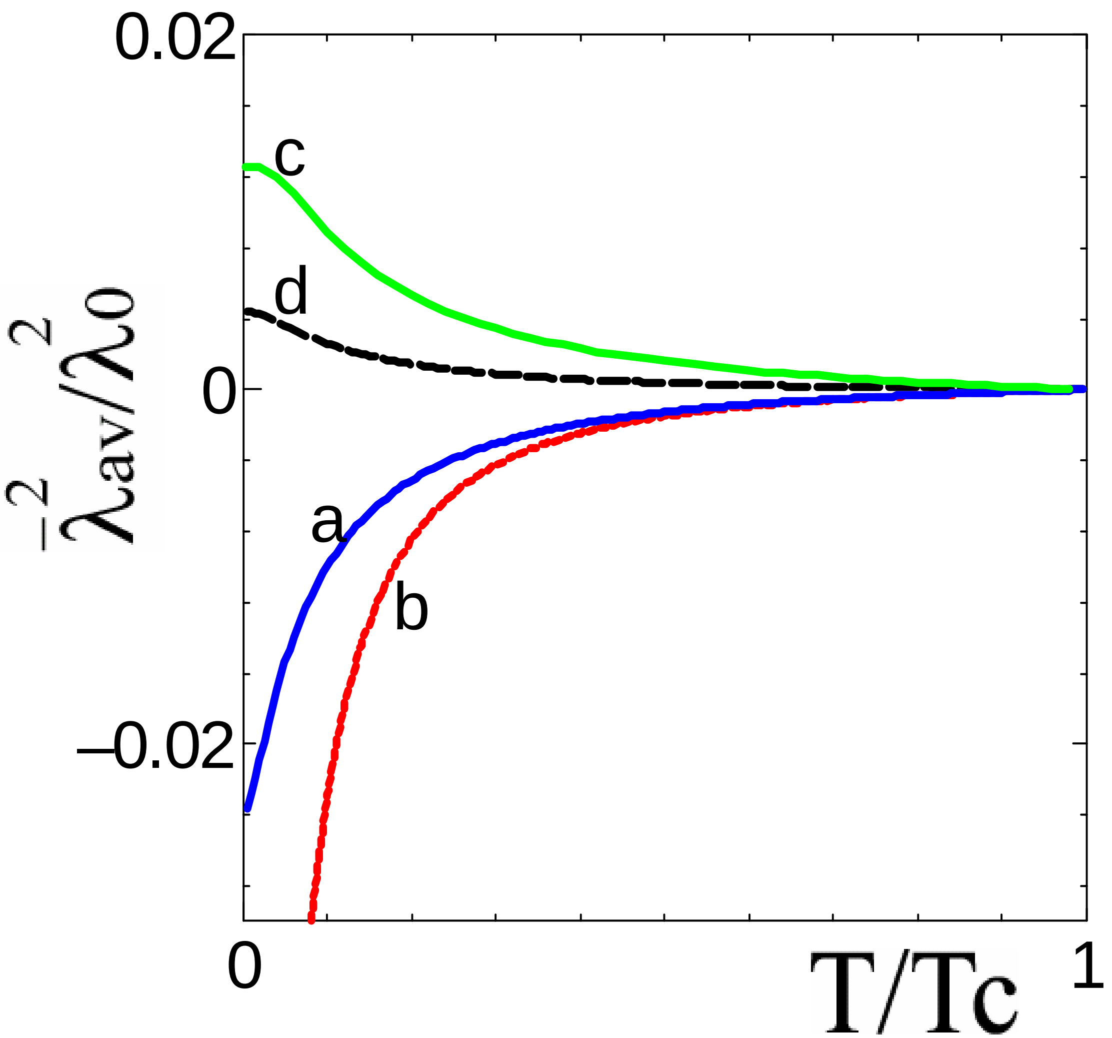
$$H(x) \sim \exp(-x/\lambda(x))$$

$$\frac{1}{\lambda^2(x)} = \frac{T\sum_{\omega_n} \sin^2\theta(\omega_n)}{\lambda_0^2}, \quad \lambda_0^{-2} = 32\pi^2 e^2 N(0) D T_C$$

$$\bar{\lambda}_{av}^2 = L / \int_0^L \frac{dx}{\lambda^2(x)}$$

Belzig, Bruder PRB 53 5727 (1996)

Temperature dependence of averaged value of local penetration depth



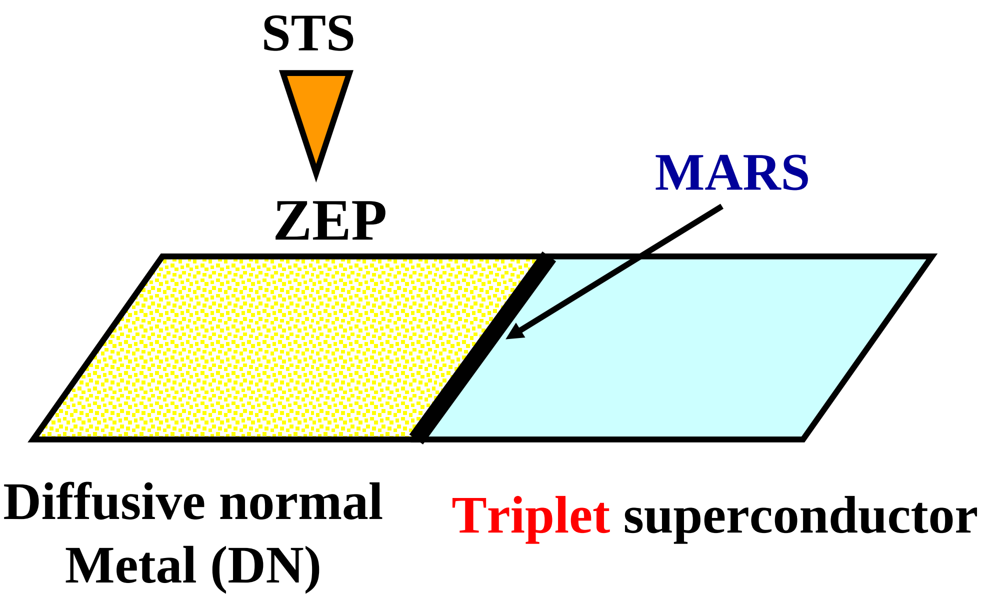
$\bar{\lambda}_{av}$ **is a purely imaginary number in the triplet case**

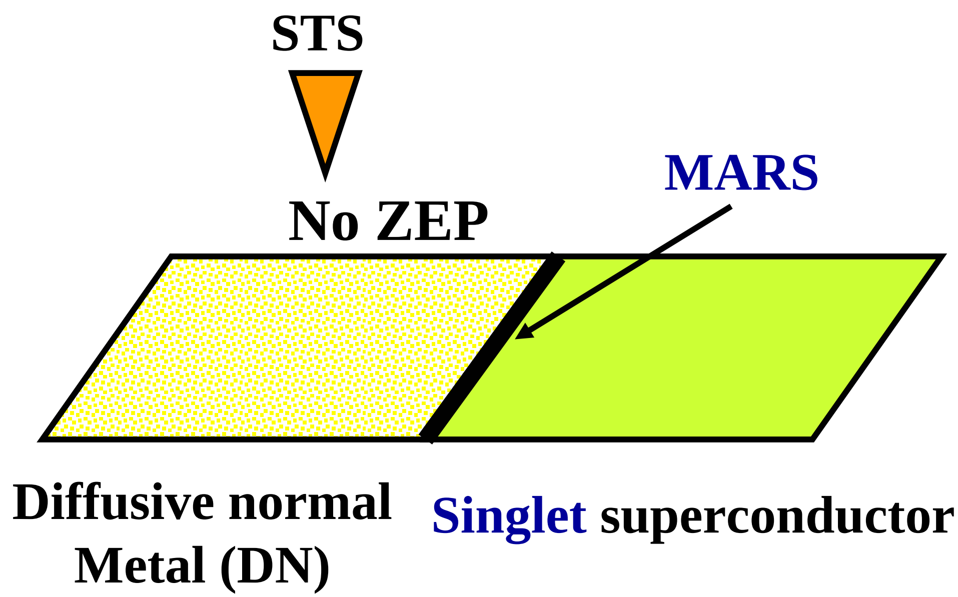
a: $p_x + ip_y$-wave
b: p_x-wave
c: s-wave
d: $d_{x^2-y^2} + id_{xy}$-wave

⇒ no screening
spatial oscillations of the magnetic field

New idea to detect triplet superconductor

MARS (Mid gap Andreev resonance state) can penetrate into DN by **proximity effect** only for triplet superconductor junctions





LDOS in DN has a zero energy peak

LDOS in DN does not have a zero energy peak

Summary

Charge transport in diffusive DN/unconventional S junctions

1. **Singlet junctions (d-wave)**
   MARS (Andreev resonant state) competes with proximity effect
   => nonmonotonous temperature dependence
   of the Josephson current in SNS junctions

2. **Triplet junctions (p-wave)**
   MARS coexist with proximity effect
   Total resistance of the junction is drastically reduced

   The pair amplitude f(*E*) has unusual E-dependence =>
   anomalies in DOS and magnetic field screening